Monetary Economics

Lecture 2: classical building blocks, part one

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This class

- Solving the classical monetary model
  
  1- real variables independent of nominal variables
  
  2- fluctuations in real variables driven by fluctuations in productivity

- Reading: Gali (2008), chapter 2 section 2.3 and appendix 2.1
Equilibrium in classical model

- A competitive equilibrium involves
  - households optimising taking prices as given
  - firms optimising taking prices as given
  - prices such that markets clear

- Optimality conditions for labor supply and demand give

\[
\frac{-U_n(C_t, N_t)}{U_c(C_t, N_t)} = \frac{W_t}{P_t} = A_t F'(N_t)
\]

- Goods market clearing

\[ Y_t = C_t \]

- Bond market clears if goods market clears
Standard functional forms

- Utility function

\[ U(C, N) = \frac{C^{1-\sigma}}{1 - \sigma} - \frac{N^{1+\varphi}}{1 + \varphi} \]

\( \sigma \) is (constant) coefficient of relative risk aversion (CRRA)

\[ \sigma = -\frac{U_{cc}(\cdot) C}{U_c(\cdot)} \]

\( 1/\sigma \) is (constant) intertemporal elasticity of substitution (IES)

\[ \frac{C^{1-\sigma} - 1}{1 - \sigma} \to \log C \quad \text{as} \ \sigma \to 1 \]

\( 1/\varphi \) is (\( \lambda \)-constant) Frisch elasticity of labor supply

- Typical numbers in macro, \( \sigma = 1 \) or 2 and \( \varphi = 0.4 \)
Standard functional forms

- Production function

\[ Y = AF(N) = AN^{1-\alpha} \]

1 $-$ $\alpha$ is labor’s (constant) share in final output

\[ 1 - \alpha = \frac{WN}{PY} \]

- Implicitly $\alpha$ share is paid to capital and other fixed factors

- Typical number in macro, $\alpha = 1/3$ so that labor’s share is 2/3
Solving the model

- With these functional forms

\[ N^\varphi C^\sigma = \frac{W}{P} = (1 - \alpha) A N^{1-\alpha} \]

- Goods market clearing

\[ C = Y = A N^{1-\alpha} \]

- Two equations in two unknowns \( C, N \) given exogenous \( A \)

- Solve for real variables independently of all nominal variables
Notation

- Little letters denote logs (or other proportional variables)
  \[ c \equiv \log C, \quad n \equiv \log N, \quad a \equiv \log A \]

- Bars denote non-stochastic steady state
  \[ \bar{c} \equiv \log \bar{C}, \quad \bar{n} \equiv \log \bar{N}, \quad \bar{a} \equiv \log \bar{A} \]
  where
  \[ \bar{A} \equiv \mathbb{E}\{A\} \]

- Hats denote log deviations
  \[ \hat{c} \equiv c - \bar{c}, \quad \hat{n} \equiv n - \bar{n}, \quad \hat{a} \equiv a - \bar{a} \]
  (approximate percentage deviation from steady state)
Solution

1. Equilibrium labor

\[ \hat{n} = \psi_{na} \hat{a}, \quad \psi_{na} \equiv \frac{1 - \sigma}{(1 - \alpha)\sigma + \alpha + \varphi} \]

2. Equilibrium output and consumption

\[ \hat{y} = \psi_{ya} \hat{a}, \quad \psi_{ya} \equiv \frac{1 + \varphi}{(1 - \alpha)\sigma + \alpha + \varphi}, \quad \hat{c} = \hat{y} \]
Interpreting the solution

- Coefficients are *elasticities*

\[
\frac{dn}{da} = \psi_{na} = \frac{1 - \sigma}{(1 - \alpha)\sigma + \alpha + \varphi}
\]

One percent change in exogenous productivity \( a \) gives \( \psi_{na} \) percent change in equilibrium labor \( n \).

- Easy to calculate volatilities

\[
\text{std}\{n\} = |\psi_{na}| \text{std}\{a\}
\]

and

\[
\text{std}\{y\} = |\psi_{ya}| \text{std}\{a\}
\]
Interpreting the solution

- Relative volatilities
  \[\frac{\text{std}\{n\}}{\text{std}\{y\}} = \frac{|\psi_{na}|}{|\psi_{ya}|} = \frac{|1 - \sigma|}{1 + \varphi}\]

- Correlations
  \[\text{corr}\{n, y\} \equiv \frac{\text{cov}\{n, y\}}{\text{std}\{n\} \text{std}\{y\}} = \frac{\psi_{na} \psi_{ya}}{|\psi_{na}| |\psi_{ya}|}\]

- Endogenous real variables $n, c, y, w - p$ inherit time series properties of exogenous productivity $a$ (autocorrelation, impulse responses, etc)
Euler equation

- Recall nominal bond prices $Q_t$ given by

$$Q_t = \mathbb{E}_t \left\{ \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

- With our standard separable preferences

$$Q_t = \mathbb{E}_t \left\{ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

- Write in terms of nominal interest rate $i_t$, inflation $\pi_t$ etc

$$i_t \equiv -\log Q_t, \quad \rho \equiv -\log \beta, \quad p_t \equiv \log P_t, \quad \pi_{t+1} \equiv p_{t+1} - p_t$$

- So Euler equation can be written

$$1 = \mathbb{E}_t \{ \exp(-\rho - \sigma \Delta c_{t+1} - \pi_{t+1} + i_t) \}$$
Log-linearised Euler equation

- Euler equation

\[ 1 = \mathbb{E}_t \{ \exp(z_{t+1}) \} \]

where

\[ z_{t+1} \equiv -\rho - \sigma \Delta c_{t+1} - \pi_{t+1} + i_t \]

- First order approximation of \( \exp(z) \) around \( z \approx 0 \) is

\[ \exp(z) \approx \exp(0) + \exp(0)(z - 0) = 1 + z \]

- Treat approximation as exact and simplify

\[ i_t = \rho + \sigma \mathbb{E}_t \{ \Delta c_{t+1} \} + \mathbb{E}_t \{ \pi_{t+1} \} \]
Euler equation and Fisher equation

- Define ex ante real interest rate $r_t$ by Fisher equation

$$r_t = i_t - \mathbb{E}_t\{\pi_{t+1}\}$$

- And since log linear Euler equation is

$$i_t = \rho + \sigma \mathbb{E}_t\{\Delta c_{t+1}\} + \mathbb{E}_t\{\pi_{t+1}\}$$

- Can eliminate nominal interest rate $i_t$ to get

$$r_t = \rho + \sigma \mathbb{E}_t\{\Delta c_{t+1}\}$$

- Familiar version of Euler equation

$$\mathbb{E}_t\{\Delta c_{t+1}\} = \frac{r_t - \rho}{\sigma}$$

(recall, $1/\sigma$ is intertemporal elasticity of substitution)
Equilibrium real interest rates

• Up to log linear approximation, real interest rates $r_t$ given by

$$r_t = \rho + \sigma \mathbb{E}_t \{ \Delta c_{t+1} \}$$

• Aside: in new Keynesian literature, this often re-written as

$$y_t = -\frac{r_t - \rho}{\sigma} + \mathbb{E}_t \{ y_{t+1} \}$$

and called the dynamic IS curve (why?)

• Equilibrium real interest rates therefore

$$r_t = \rho + \sigma \psi_{ya} \mathbb{E}_t \{ \Delta a_{t+1} \}$$
Equilibrium real interest rates

• Conditional expectation determined by productivity process

• AR(1) example

\[ a_{t+1} = \phi_a a_t + \varepsilon_{a,t+1} \quad \varepsilon_{a,t+1} \sim \text{IID and } N(0, \sigma_{\varepsilon}^2) \]

with persistence \(0 < \phi_a < 1\)

Then

\[ r_t = \rho - \sigma \psi y_a (1 - \phi_a) a_t \]

moves in opposite direction to productivity, since \(\phi_a < 1\)

• All interesting real variables now determined, all independent of nominal variables
Next class

- Money demand and price level determination
- Reading: Gali (2008), chapter 2 sections 2.4–2.5