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ECON40002
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Advanced Macroeconomics
Tutorial #7

Stochastic growth model in Dynare, nonstationary version. Consider a stochastic growth model with constant labor force $L > 0$ where output is given by the production function

$$Y_t = K_t^{\alpha}(A_t L)^{1-\alpha}, \qquad 0 < \alpha < 1$$

Let $g_t \equiv A_t / A_{t-1}$ denote the gross growth rate of labor-augmenting productivity and suppose that this growth rate follows a stationary AR(1) process in logs

$$\log g_{t+1} = (1-\phi)\log \bar{g} + \phi \log g_t + \varepsilon_{t+1}, \qquad 0 < \phi < 1, \qquad \bar{g} \geq 1$$

where the innovations ε_t are IID $N(0, \sigma_\varepsilon^2)$. For the sake of interpretation, note that $\log g_t = \Delta \log A_t$, i.e., is approximately the net growth rate. Now suppose that the planner maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C_t/L)\, L, \qquad 0 < \beta < 1$$

where C_t/L denotes consumption per worker, subject to the resource constraints

$$C_t + K_{t+1} = K_t^{\alpha}(A_t L)^{1-\alpha} + (1-\delta)K_t, \qquad 0 < \delta < 1$$

given initial conditions $K_0, A_0 > 0$ and the process for productivity growth above.

(a) Let $c_t = C_t/A_t L$ and $k_{t+1} = K_{t+1}/A_t L$ etc denote the detrended levels of consumption, capital etc. Derive the planner's key optimality conditions for c_t and k_{t+1}.

(b) Solve for the non-stochastic steady state values of the detrended levels $\bar{c}$ and $\bar{k}$. What is the effect of an increase in the long-run average growth rate $\bar{g}$ on $\bar{c}$ and $\bar{k}$? Explain.

Now suppose the following parameter values: $\alpha = 0.3$, $\beta = 0.98$, $\delta = 0.02$ and for the growth process $\bar{g} = 1.02$, $\phi = 0$ and $\sigma_\varepsilon = 0.015$. Use Dynare to do the following:

(c) Calculate the long-run standard deviations of productivity growth and the detrended levels of consumption, capital, output and investment. Which of these variables move most closely together? Which of these variables is most volatile? Explain.

(d) Suppose the economy is at steady state and that at $t = 0$ there is a 1 standard deviation innovation to productivity growth, i.e., $\varepsilon_0 = \sigma_\varepsilon$. Calculate and plot for $T = 50$ periods after the shock the impulse response functions for both (i) the detrended levels of consumption, capital, output and investment, and (ii) the growth rates $\Delta \log C_t$, $\Delta \log K_t$, $\Delta \log Y_t$, $\Delta \log I_t$ and $\Delta \log A_t$. Give intuition for your findings.

(e) Now let $K_0 = A_0 = 1$ and simulate a sequence of productivity growth $\log g_t$ of length $T = 250$ and use this to generate simulated sequences of $\log C_t$, $\log K_t$, $\log Y_t$ and $\log A_t$. Explain the dynamics of these variables and the dynamics of the ratios $\log(C_t/Y_t)$, and $\log(K_t/Y_t)$. Give intuition for your findings.

(f) Suppose instead that $\phi = 0.5$. How if at all do your answers to (c), (d) and (e) change? Explain.