316-632 INTERNATIONAL MONETARY ECONOMICS NOTE 7b

Chris Edmond ⟨cpedmond@unimelb.edu.au⟩

Exchange rate risk premia?

Recall that if covered interest parity held in data, we would have something like

$$\begin{aligned} i_t - i_t^* &= \mathsf{E}_t\{\Delta e_{t+1}\} \\ f_t &= \mathsf{E}_t\{e_{t+1}\} \end{aligned}$$

so that interest differentials are explained by expected depreciations and the one-period log forward rate is the expected log spot rate in one period's time. However, and as discussed in class, if we run a regression of the form

$$e_{t+1} - e_t = \beta_0 + \beta_1(f_t - e_t) + \text{noise}$$

a typical estimate is $\hat{\beta}_1 = -0.88$ or thereabouts, not the $\hat{\beta}_1 = 1$ that we expect from the uncovered interest parity hypothesis. Put differently, in the data $i_t > i_t^*$ goes hand-in-hand with exchange rate **appreciations**, $\Delta e_{t+1} < 0$.

If we define an exchange rate **risk premium** by

$$\rho_t \equiv f_t - \mathsf{E}_t\{e_{t+1}\}$$

we have the decomposition

$$\begin{aligned} f_t - e_t &= f_t - \mathsf{E}_t\{e_{t+1}\} + \mathsf{E}_t\{e_{t+1}\} - e_t \\ &= \rho_t + \mathsf{E}_t\{\Delta e_{t+1}\} \end{aligned}$$

then we can do a little bit of work to figure out the statistical properties of the risk premium that would give rise to $\hat{\beta}_1 < 0$. The following discussion essentially follows Fama (1984). Notice that the use of the term "risk premium" is a little bit misleading: at the moment we're really just giving a label to our ignorance.

- *Basic econometrics*: Recall that the probability limit of the slope coefficient is

$$\beta_1 = \frac{\mathsf{Cov}\{f_t - e_t, \Delta e_{t+1}\}}{\mathsf{Var}\{f_t - e_t\}}$$

- *Rational expectations*: Rational expectations requires that $\mathsf{E}_t\{e_{t+1}\} - e_{t+1}$ must be uncorrelated with any information that is observable at date t. Because of this, we have have

$$\mathsf{Cov}\{f_t - e_t, \Delta e_{t+1}\} = \mathsf{Cov}\{f_t - e_t, \mathsf{E}_t\{\Delta e_{t+1}\}\}$$

  Hence
$$\beta_1 = \frac{\mathsf{Cov}\{f_t - e_t, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{f_t - e_t\}} = \frac{\mathsf{Cov}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}\}}$$

- *Properties of covariances*: If we have two random variables X and Y, then

$$\mathsf{Cov}\{X, X+Y\} = \mathsf{Var}\{X\} + \mathsf{Cov}\{X, Y\}$$

  So

$$\beta_1 = \frac{\mathsf{Cov}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}\}} = \frac{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}\}}$$

- Several implications follow from this calculation. First, if the risk premium were constant, $\rho_t = \rho$ all t, then $\mathsf{Var}\{\rho_t\} = 0$ and we would have slope coefficient

$$\beta_1 = \frac{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\}} = 1$$

  So in order for the slope coefficient to be other than one, we definitely have to have a **time-varying risk premium**. Also, since $\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} \geq 0$, in order for us to have $\beta_1 < 0$ we must have a risk premium with the property

$$\mathsf{Cov}\{\mathsf{E}_t\{\Delta e_{t+1}\}, \rho_t\} < 0$$

  That is, there must be a systematic tendency for expected **appreciations**, $\mathsf{E}_t\{\Delta e_{t+1}\} < 0$, to go hand-in-hand with **increases** in the risk premium. Moreover, the negative covariance has to have magnitude such that

$$|\mathsf{Cov}\{\mathsf{E}_t\{\Delta e_{t+1}\}, \rho_t\}| > \mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\}$$

  As we will see below, this requirement will turn out not to be particularly stringent, so the literature does not focus on it much.

- Another implication for the statistical properties of the risk premium comes from expanding the denominator of β_1 so that we have

$$\begin{aligned}\beta_1 &= \frac{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\rho_t + \mathsf{E}_t\{\Delta e_{t+1}\}\}} \\ &= \frac{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\}}{\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + 2\mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Var}\{\rho_t\}}\end{aligned}$$

  Now recall that a typical estimate for β_1 is negative. But whenever $\beta_1 < \frac{1}{2}$, we must have

$$\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\} < \frac{1}{2}\left[\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + 2\mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Var}\{\rho_t\}\right]$$

  or

$$\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} < \frac{1}{2}\left[\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} + \mathsf{Var}\{\rho_t\}\right]$$

  In short

$$\mathsf{Var}\{\rho_t\} > \mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\}$$

  Hence the risk premium must be relatively volatile in comparison with expected depreciations.

- To sum up: in order to get a negative slope coefficient in the forward premium regression under rational expectations, we must have a risk premium that is relatively volatile (compared to the expected depreciation) and that co-varies negatively with the expected depreciation.

- The requirement that the risk premium co-vary negatively with expected depreciations is particularly counterintuitive if one thought of certain currencies as being safe-havens during times of distress in global financial markets. That is, if there are "flights to quality", maybe one ought to expect currencies to appreciate when their risk premium goes down! (This would imply that $\mathsf{Cov}\{\rho_t, \mathsf{E}_t\{\Delta e_{t+1}\}\} > 0$).

- It is also worth bearing in mind that nominal exchange rate movements are well approximated by random walks (see Meese and Rogoff, JIE 1983) so $\mathsf{E}_t\{e_{t+1}\} \simeq a + e_t$ (some constant a) and $\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} \simeq 0$. So the forward premium puzzle may not really be that the volatility of the risk premium is so large but instead that the volatility of expected depreciations is **so small**.

- Notice also that if $\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} = 0$ **nearly all of the observed variations in interest differentials are accounted for by the movements in the "risk premium"**. That is, if $\mathsf{Var}\{\mathsf{E}_t\{\Delta e_{t+1}\}\} = 0$, we have

$$\begin{aligned} i_t - i_t^* &= f_t - e_t \\ &= f_t - \mathsf{E}_t\{e_{t+1}\} + \mathsf{E}_t\{e_{t+1}\} - e_t \\ &= \rho_t + \mathsf{E}_t\{\Delta e_{t+1}\} \\ &\simeq \rho_t + a \end{aligned}$$

and so

$$\mathsf{Var}\{i_t - i_t^*\} \simeq \mathsf{Var}\{\rho_t\}$$

Chris Edmond

12 September 2004